Question	Scheme	Marks	AOs
1(a)	$V=\pi r^2h=355\Rightarrow h=\dfrac{355}{\pi r^2}$ $\left(\text{or } rh=\dfrac{355}{\pi r} \text{ or } \pi rh=\dfrac{355}{r}\right)$	B1	1.1b
	$C=0.04\left(\pi r^2+2\pi rh\right)+0.09\left(\pi r^2\right)$	M1	3.4
	$C=0.13\pi r^2+0.08\pi rh=0.13\pi r^2+0.08\pi r\left(\dfrac{355}{\pi r^2}\right)$	dM1	2.1
	$C=0.13\pi r^2+\dfrac{28.4}{r}$ *	A1*	1.1b
		(4)	
(b)	$\dfrac{dC}{dr}=0.26\pi r-\dfrac{28.4}{r^2}$	M1 A1	3.4 1.1b
	$\dfrac{dC}{dr}=0\Rightarrow r^3=\dfrac{28.4}{0.26\pi}\Rightarrow r=...$	M1	1.1b
	$r=\sqrt[3]{\dfrac{1420}{13\pi}}=3.26...$	A1	1.1b
		(4)	
(c)	$\left(\dfrac{d^2C}{dr^2}=\right)\ 0.26\pi+\dfrac{56.8}{r^3}=0.26\pi+\dfrac{56.8}{"3.26"^3}$	M1	1.1b
	$\left(\dfrac{d^2C}{dr^2}=\right)\ (2.45..)>0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C=0.13\pi("3.26")^2+\dfrac{28.4}{"3.26"}$	M1	3.4
	$(C=)13$	A1	1.1b
		(2)	
		(12 marks)	

Notes

(a)

B1: Correct expression for h or rh or πrh in terms of r. This may be implied by their later substitution.

M1: Scored for the sum of the three terms of the form $0.04...r^2$, $0.09...r^2$ and $0.04\times...rh$

The $0.04\times...rh$ may be implied by eg $0.04\times...r\times\dfrac{355}{\pi r^2}$ if h has already been replaced

dM1: Substitutes h or rh or πrh into their equation for C which must be of an allowable form (see above) to obtain an equation connecting C and r.
It is dependent on a correct expression for h or rh or πrh in terms of r

A1*: Achieves given answer with no errors. Allow Cost instead of C but they cannot just have an expression.
As a minimum you must see
- the separate equation for volume
- the two costs for the top and bottom separate before combining
- a substitution before seeing the $\frac{28.4}{r}$ term

Eg $355 = \pi r^2 h$ and $C = 0.04\pi r^2 + 0.09\pi r^2 + 0.04 \times 2\pi rh = 0.13\pi r^2 + 0.08\pi \times \left(\frac{355}{\pi r}\right)$

(b)

M1: Differentiates to obtain at least $r^{-1} \rightarrow r^{-2}$

A1: Correct derivative.

M1: Sets $\frac{dC}{dr} = 0$ and solves for r. There must have been some attempt at differentiation of the equation for C ($...r^2 \rightarrow ...r$ or $...r^{-1} \rightarrow ...r^{-2}$) Do not be concerned with the mechanics of their rearrangement and do not withhold this mark if their solution for r is negative

A1: Correct value for r. Allow exact value or awrt 3.26

(c)

M1: Finds $\frac{d^2C}{dr^2}$ at their (positive) r or considers the sign of $\frac{d^2C}{dr^2}$.

This mark can be scored as long as their second derivative is of the form $A + \frac{B}{r^3}$ where A and B are non zero

A1: Requires
- A correct $\frac{d^2C}{dr^2}$
- Either
    - deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$ (without evaluating). There must be some minimal explanation as to why it is positive.
    - substitute their positive r into $\frac{d^2C}{dr^2}$ without evaluating and deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$
    - evaluate $\frac{d^2C}{dr^2}$ (which must be awrt 2.5) and deduces $\frac{d^2C}{dr^2} > 0$ for $r > 0$

(d)

M1: Uses the model and their positive r found in (b) to find the minimum cost. Their r embedded in the expression is sufficient. May be seen in (b) but must be used in (d).

A1: $(C =)\ 13$ ignore units

Question	Scheme	Marks	AOs
2 (a)	$\left\{\frac{dy}{dx}=\right\}2x^2-7x-4$	M1 A1	1.1b 1.1b
		(2)	
(b)	Attempts to solve $\left\{\frac{dy}{dx}=\right\}2x^2-7x-4\ldots0$ e.g., $(2x+1)(x-4)=0$ leading to $x=\ldots$ and $x=\ldots$	M1	1.1b
	Correct critical values $x=-\frac{1}{2},4$	A1	1.1b
	Chooses inside region for their critical values	dM1	1.1b
	Accept either $-\frac{1}{2}<x<4$ **or** $-\frac{1}{2}\le x\le 4$	A1	1.1b
		(4)	
		(6 marks)	

Notes:

(a)

M1: Decreases the power of x by one for at least one of their terms. Look for $x^n \to \ldots x^{n-1}$

Allow for $5\to 0$

A1: $\left\{\frac{dy}{dx}=\right\}2x^2-7x-4$

(b)

M1: Sets their $\frac{dy}{dx}\ldots0$ where … may be an equality or an inequality and proceeds to find two values for x from a 3TQ using the usual rules. This may be implied by their critical values.

A1: Correct critical values $x\ldots-\frac{1}{2},4$

These may come directly from a calculator and might only be seen on a sketch.

dM1: Chooses the inside region for their critical values.

A1: Accept either $-\frac{1}{2}<x<4$ **or** $-\frac{1}{2}\le x\le 4$ but not, e.g., $-\frac{1}{2}<x\le 4$

Condone, e.g., $x>-\frac{1}{2},\ \ x<4$ **or** $x>-\frac{1}{2}$ and $x<4$ **or** $\left\{x:x>-\frac{1}{2}\right\}\cap\{x:x<4\}$

or $x\in\left(-\frac{1}{2},4\right)$ **or** $x\in\left[-\frac{1}{2},4\right]$

Note: You may see $x<-\frac{1}{2},x<4$ in their initial work before $-\frac{1}{2}<x<4$. Condone this so long as it is clear that the $-\frac{1}{2}<x<4$ is their final answer.

Question	Scheme	Marks	AOs
3	$y=\dfrac{x-4}{2+\sqrt{x}} \Rightarrow \dfrac{dy}{dx}=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$=\dfrac{2+\sqrt{x}-(x-4)\frac{1}{2}x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2+\sqrt{x}-\frac{1}{2}\sqrt{x}+2x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}=\dfrac{2\sqrt{x}+\frac{1}{2}x+2}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$	M1	1.1b
	$=\dfrac{x+4\sqrt{x}+4}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{\left(2+\sqrt{x}\right)^2}{2\sqrt{x}\left(2+\sqrt{x}\right)^2}=\dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
		(4 marks)	

Notes

M1: Attempts to use a correct rule e.g. quotient or product (& chain) rule to achieve the following forms

Quotient : $\dfrac{\alpha\left(2+\sqrt{x}\right)-\beta(x-4)x^{-\frac{1}{2}}}{\left(2+\sqrt{x}\right)^2}$ but be tolerant of attempts where the $\left(2+\sqrt{x}\right)^2$ has been incorrectly expanded

Product: $\alpha\left(2+\sqrt{x}\right)^{-1}+\beta x^{-\frac{1}{2}}(x-4)\left(2+\sqrt{x}\right)^{-2}$

Alternatively with $t=\sqrt{x}$, $y=\dfrac{t^2-4}{2+t}\Rightarrow\dfrac{dy}{dx}=\dfrac{dy}{dt}\times\dfrac{dt}{dx}=\dfrac{2t(2+t)-\left(t^2-4\right)}{(2+t)^2}\times\dfrac{1}{2}x^{-\frac{1}{2}}$ with same rules

A1: Correct derivative in any form. Must be in terms of a single variable (which could be t)

M1: Following a correct attempt at differentiation, it is scored for multiplying both numerator and denominator by $\sqrt{x}$ and collecting terms to form a single fraction. It can also be scored from $\dfrac{uv'-vu'}{v^2}$

For the $t=\sqrt{x}$, look for an attempt to simplify $\dfrac{t^2+4t+4}{(2+t)^2}\times\dfrac{1}{2t}$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
3	$y=\dfrac{x-4}{2+\sqrt{x}}\Rightarrow y=\dfrac{\left(\sqrt{x}+2\right)\left(\sqrt{x}-2\right)}{2+\sqrt{x}}=\sqrt{x}-2$	M1 A1	2.1 1.1b
	$\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{x}}$	M1 A1	1.1b 2.1
		(4)	
		(4 marks)	

Notes

M1: Attempts to use difference of two squares. Can also be scored using

$t=\sqrt{x}\Rightarrow y=\dfrac{t^2-4}{t+2}\Rightarrow y=\dfrac{(t+2)(t-2)}{t+2}$

A1: $y=\sqrt{x}-2$ or $y=t-2$

M1: Attempts to differentiate an expression of the form $y=\sqrt{x}+b$

A1: Correct expression showing all key steps with no errors or omissions. $\dfrac{dy}{dx}$ must be seen at least once

Question	Scheme	Marks	AOs
4(a)(i)	$\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 60x^2 - 144x + 84$	A1ft	1.1b
		(3)	
(b)(i)	$x = 1 \Rightarrow \frac{dy}{dx} = 20 - 72 + 84 - 32$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 1$	A1	2.1
	Alternative for (b)(i)		
	$20x^3 - 72x^2 + 84x - 32 = 4(x-1)^2(5x-8) = 0 \Rightarrow x = ...$	M1	1.1b
	When $x = 1$, $\frac{dy}{dx} = 0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8} = ...$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8} > 0,$ $\left(\frac{d^2y}{dx^2}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 60x^2 - 144x + 84 = 0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right) = 120x - 144 \Rightarrow \left(\frac{d^3y}{dx^3}\right)_{x=1} = ...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1} = 0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1} \neq 0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8} = ...$ $\left(\frac{dy}{dx}\right)_{x=1.2} = ...$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8} < 0,$ $\left(\frac{dy}{dx}\right)_{x=1.2} < 0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)
M1: $x^n \to x^{n-1}$ for at least one power of x
A1: $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$
(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(b)(i)

M1: Substitutes x = 1 into their $\frac{dy}{dx}$

A1: Obtains $\frac{dy}{dx} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx} = 0$ and then shows $\frac{dy}{dx} = 0$

Alternative:

M1: Attempts to solve $\frac{dy}{dx} = 0$ by factorisation. This may be by using the factor of (x – 1) or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and x = 1 seen as the roots.

A1: Obtains x = 1 and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of x = 1. Do not be too concerned with the interval for the method mark.

(NB $\frac{d^2y}{dx^2} = (x-1)(60x-84)$ so may use this factorised form when considering $x < 1$, $x > 1$ for sign change of second derivative)

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where x < 1.4

Alternative 1 for (b)(ii)

M1: Shows that second derivative at x = 1 is zero and **then finds the third derivative at x = 1**

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "≠ 0" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of x = 1. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where x < 1.4

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f′(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f″(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f′(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f″(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

Question	Scheme	Marks	AOs
5 (a)	$2 < x < 6$	B1	1.1b
		(1)	
(b)	States either $k > 8$ or $k < 0$	M1	3.1a
	States e.g. $\{k : k > 8\} \cup \{k : k < 0\}$	A1	2.5
		(2)	
(c)	**Please see notes for alternatives**		
	States $y = ax(x-6)^2$ or $\mathrm{f}(x) = ax(x-6)^2$	M1	1.1b
	Substitutes $(2,8)$ into $y = ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y = \frac{1}{4}x(x-6)^2$ or $\mathrm{f}(x) = \frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
			(6 marks)
Notes: Watch for answers written by the question. If they are beside the question and in the answer space, the one in the answer space takes precedence			

(a)

B1: Deduces $2 < x < 6$ o.e. such as $x > 2, x < 6$ $\quad x > 2$ and $x < 6$ $\quad \{x : x > 2\} \cap \{x : x < 6\}$ $\quad x \in (2,6)$

Condone attempts in which set notation is incorrectly attempted but correct values can be seen or implied E.g. $\{x > 2\} \cap \{x < 6\}$ $\{x > 2, x < 6\}$. Allow just the open interval $(2,6)$

Do not allow for incorrect inequalities such as e.g. $x > 2$ or $x < 6$, $\{x : x > 2\} \cup \{x : x < 6\}$ $x \in [2,6]$

(b)

M1: Establishes a correct method by finding one of the (correct) inequalities

States either $k > 8$ (condone $k \geqslant 8$) or $k < 0$ (condone $k \leqslant 0$)

Condone for this mark $y \leftrightarrow k$ or $\mathrm{f}(x) \leftrightarrow k$ and $8 < k < 0$

A1: Fully correct solution in the form $\{k : k > 8\} \cup \{k : k < 0\}$ or $\{k | k > 8\} \cup \{k | k < 0\}$ either way around but condone $\{k < 0\} \cup \{k > 8\}$, $\{k : k < 0 \cup k > 8\}$, $\{k < 0 \cup k > 8\}$. It is not necessary to mention $\mathbb{R}$, e.g. $\{k : k \in \mathbb{R}, k > 8\} \cup \{k : k \in \mathbb{R}, k < 0\}$ Look for $\{\ \}$ and $\cup$

Do not allow solutions not in set notation such as $k < 0$ or $k > 8$.

(c)

M1: Realises that the equation of C is of the form $y = ax(x-6)^2$. Condone with $a = 1$ for this mark.

So award for sight of $ax(x-6)^2$ even with $a = 1$

dM1: Substitutes $(2,8)$ into the form $y = ax(x-6)^2$ and attempts to find the value for a.

It is dependent upon having an equation, which the (y =..) may be implied, of the correct form.

A1: Uses all of the information to form a correct **equation** for C $\quad y = \frac{1}{4}x(x-6)^2$ o.e.

ISW after a correct answer. Condone $\mathrm{f}(x) = \frac{1}{4}x(x-6)^2$ but not $C = \frac{1}{4}x(x-6)^2$.

Allow this to be written down for all 3 marks

Examples of alternative methods

....

Alternative I part (c):
Using the form $y = ax^3 + bx^2 + cx$ and setting up then solving simultaneous equations.
There are various versions of this but can be marked similarly

M1: Realises that the equation of *C* is of the form $y = ax^3 + bx^2 + cx$ and forms two equations in *a*, *b* and *c*. Condone with *a* =1 for this mark.
Note that the form $y = ax^3 + bx^2 + cx + d$ is M0 until *d* is set equal to 0.
There are four equations that could be formed, only two are necessary for this mark.
Condone slips
Using (6, 0) $\Rightarrow 216a+36b+6c=0$
Using (2, 8) $\Rightarrow 8a+4b+2c=8$
Using $\frac{dy}{dx}=0$ at $x = 2$ $\Rightarrow 12a+4b+c=0$
Using $\frac{dy}{dx}=0$ at $x = 6 \Rightarrow 108a+12b+c=0$

dM1: Forms and solves three different equations, one of which must be using (2, 8) to find values for *a*, *b* and *c*. A calculator can be used to solve the equations

A1: Uses all of the information to form a correct equation for *C* $y = \frac{1}{4}x^3 - 3x^2 + 9x$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{1}{4}x^3 - 3x^2 + 9x$

....

Alternative II part (c)
Using the gradient and integrating

M1: Realises that the gradient of *C* is zero at 2 and 6 so sets $f'(x) = k(x-2)(x-6)$ oe **and** attempts to integrate. Condone with *k* = 1

dM1: Substitutes $x = 2, y = 8$ into $f(x) = k\left(...x^3 + ...x + ...\right)$ and finds a value for *k*

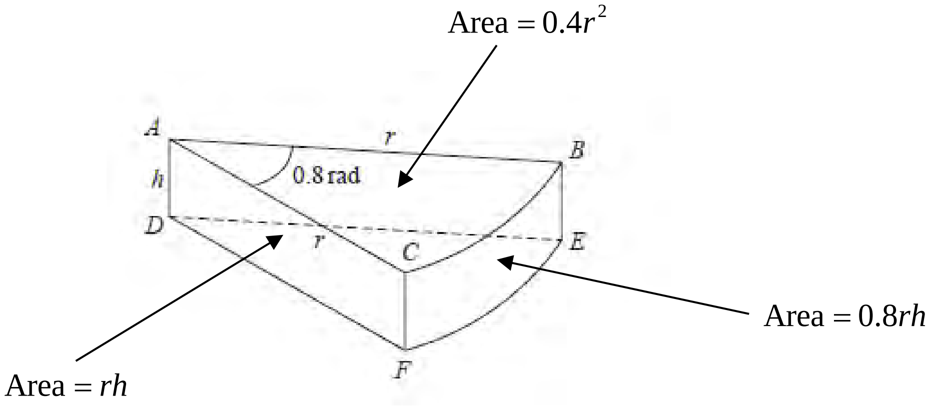
A1: Uses all of the information to form a correct equation for *C* $y = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$ o.e.

ISW after a correct answer. Condone $f(x) = \frac{3}{4}\left(\frac{1}{3}x^3 - 4x^2 + 12x\right)$

....

Question	Scheme	Marks	AOs
6 (a)	Sets up an allowable equation using volume $= 240$ E.g. $\frac{1}{2}r^2 \times 0.8h = 240 \Rightarrow h = \frac{600}{r^2}$ o.e.	M1 A1	3.4 1.1b
	Attempts to substitute their $h = \frac{600}{r^2}$ into $(S =)\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + 2rh + 0.8rh$	dM1	3.4
	$S = 0.8r^2 + 2.8rh = 0.8r^2 + 2.8\times\frac{600}{r} = 0.8r^2 + \frac{1680}{r}$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$	M1 A1	3.1a 1.1b
	Sets $\frac{dS}{dr} = 0 \Rightarrow r^3 = 1050$ $r =$ awrt 10.2	dM1 A1	2.1 1.1b
		(4)	
(c)	Attempts to substitute their positive r into $\left(\frac{d^2S}{dr^2}\right) = 1.6 + \frac{3360}{r^3}$ and considers its value or sign	M1	1.1b
	E.g. Correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$ with $\left.\frac{d^2S}{dr^2}\right\|_{r=10.2} = 5 > 0$ proving a minimum value of S	A1	1.1b
		(2)	
			(10 marks)
Notes:			

Volume $= 0.4r^2h$



Total surface area = $2rh + 0.8r^2 + 0.8rh$

(a)

M1: Attempts to use the fact that the volume of the toy is 240 cm^3

Sight of $\frac{1}{2}r^2 \times 0.8 \times h = 240$ leading to $h = ...$ or $rh = ...$ scores this mark

But condone an equation of the correct form so allow for $kr^2h = 240 \Rightarrow h = ...$ or $rh = ...$

A1: A correct expression for $h = \frac{600}{r^2}$ or $rh = \frac{600}{r}$ which may be left unsimplified.

This may be implied when you see an expression for S or part of S E.g $2rh = 2r \times \frac{600}{r^2}$

dM1: Attempts to substitute their $h = \frac{a}{r^2}$ o.e. such as $hr = \frac{a}{r}$ into a **correct** expression for S

Sight of $\frac{1}{2}r^2 \times 0.8 + \frac{1}{2}r^2 \times 0.8 + rh + rh + 0.8rh$ with an appropriate substitution

Simplified versions such as $0.8r^2 + 2rh + 0.8rh$ used with an appropriate substitution is fine.

A1*: Correct work leading to the given result.

$S =$, $SA =$ or surface area = must be seen at least once in the correct place

The method must be made clear so expect to see evidence. For example

$S = 0.8r^2 + 2rh + 0.8rh \Rightarrow S = 0.8r^2 + 2r \times \frac{600}{r^2} + 0.8r \times \frac{600}{r^2} \Rightarrow S = 0.8r^2 + \frac{1680}{r}$ would be fine.

(b) There is no requirement to see $\frac{dS}{dr}$ in part (b). It may even be called $\frac{dy}{dx}$.

M1: Achieves a derivative of the form $pr \pm \frac{q}{r^2}$ where p and q are non- zero constants

A1: Achieves $\left(\frac{dS}{dr}\right) = 1.6r - \frac{1680}{r^2}$

dM1: Sets or implies that their $\frac{dS}{dr} = 0$ and proceeds to $mr^3 = n, \quad m \times n > 0$. It is dependent upon a correct attempt at differentiation. This mark may be implied by a correct answer to their $pr - \frac{q}{r^2} = 0$

A1: $r =$ awrt 10.2 or $\sqrt[3]{1050}$

(c)

M1: Attempts to substitute their positive r (found in (b)) into $\left(\frac{d^2S}{dr^2} = \right) e \pm \frac{f}{r^3}$ where e and f are non zero and finds its value or sign.

Alternatively considers the sign of $\left(\frac{d^2S}{dr^2} = \right) e \pm \frac{f}{r^3}$ (at their positive r found in (b))

Condone the $\frac{d^2S}{dr^2}$ to be $\frac{d^2y}{dx^2}$ or being absent, but only for this mark.

A1: States that $\frac{d^2S}{dr^2}$ or $S'' = 1.6 + \frac{3360}{r^3} =$ awrt $5 > 0$ proving a minimum value of S

This is dependent upon having achieved $r =$ awrt 10 and a correct $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3}$

It can be argued without finding the value of $\frac{d^2S}{dr^2}$. E.g. $\frac{d^2S}{dr^2} = 1.6 + \frac{3360}{r^3} > 0$ as $r > 0$, so minimum value of S. For consistency it is also dependent upon having achieved $r =$ awrt 10

Do **NOT** allow $\frac{d^2y}{dx^2}$ for this mark

Question	Scheme	Marks	AOs
7(a)	$\{f'(x)=\}...x^2+...x+...\Rightarrow\{f''(x)=\}...x+...$	M1	1.1b
	$\{f'(x)=\}3x^2+4x-8\Rightarrow\{f''(x)=\}6x+4$	A1cso	1.1b
		(2)	
(b)(i)	$"6x+4"=0\Rightarrow x="-\frac{2}{3}"$	B1ft	1.1b
(ii)	$x\leqslant"-\frac{2}{3}"$ or $x<"-\frac{2}{3}"$	B1ft	2.2a
		(2)	
			(4 marks)

Notes

(a)

M1: For attempting to differentiate twice.

It can be scored for any of: $x^3\to...x^2\to....x$ or $2x^2\to...x\to k$ or $-8x\to k\to 0$ where … are constants.

You can ignore the lhs so do not be concerned what they call the first and/or second derivative, just look for their expressions.

The indices do not need to be processed for this mark so allow for e.g. $x^3\to...x^{3-1}\to....x^{3-1-1}$

A1cso: $(f''(x)=)\ 6x+4$ Correct second derivative from fully correct work. The $"f''(x)="$ is not required.

Allow $6x^1$ for $6x$ but not $4x^0$ for 4 unless the $4x^0$ becomes 4 later, e.g. in part (b).

Do **not** apply isw so mark their final answer. E.g. if $6x+4$ becomes $3x+2$ score A0

(b)

(i)

B1ft: $ax+b=0\Rightarrow(x=)-\frac{b}{a}$. This mark is for obtaining $x=-\frac{2}{3}$ **or** $x=-\frac{b}{a}$ which has come from solving an equation of the form $ax+b,\ a,b\neq 0$ where $ax+b$ is their attempt to differentiate twice in part (a)

Allow equivalent fractions e.g. $x=-\frac{4}{6}$ or equivalents for their $x=-\frac{b}{a}$ or an exact decimal and isw.

(ii)

B1ft: Deduces $x\leqslant-\frac{2}{3}$ **or** follow through their single value of x from part (i) obtained from their attempt to solve an equation of the form $ax+b=0,\ a,b\neq 0$ where $ax+b$ was their attempt to differentiate twice in part (a) . Do not isw and mark their final answer.

If 2 inequalities are given e.g. $x<"-\frac{2}{3}",\ x>"-\frac{2}{3}"$ without indicating which is their answer score B0

Condone $<$ for $\leqslant$ and allow equivalent inequalities e.g. $-\frac{2}{3}>x$

Allow equivalent fractions e.g. $x=-\frac{4}{6}$ or equivalents for their $x=-\frac{b}{a}$

Allow equivalent notation so these are all acceptable:

$x\leqslant"-\frac{2}{3}",\ x<"-\frac{2}{3}",\ \left(-\infty,"-\frac{2}{3}"\right],\ \left(-\infty,"-\frac{2}{3}"\right),\ \left\{x:x\leqslant"-\frac{2}{3}"\right\},\ \left\{x:x<"-\frac{2}{3}"\right\}$

Ignore any reference to values of y.

Allow ft decimal answers from (i) which may be inexact.

Correct answers in part (b) with no working in (a) can score 0011.